Report 1528



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# DAVID TAYLOR MODEL BASIN

HYDROMECHANICS

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## THE COMPLETE EXPRESSIONS FOR "ADDED MASS" OF A RIGID BODY MOVING IN AN IDEAL FLUID

by

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HYDROMECHANICS LABORATORY

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## NOTATION

In order to conform to related work at the David Taylor Model Basin, the nomenclature given in Reference 1 (see page 19) has been employed. Pertinent items of the notation are listed here for convenient reference, and are also defined in the text at the point where first used.

| | |
|---|---|
| $a$ | Linear acceleration of the center of mass of the body |
| $a, b, c$ | Constants defining the semiaxes of an ellipsoid |
| $e$ | Eccentricity of the meridian elliptical section of a prolate spheroid |
| $F$ | Total external force acting on the body |
| $F_1$ | That part of F which is used to change the kinetic energy of the fluid when the body is accelerated |
| $F_2$ | Vector difference $F - F_1$ |
| $K_1, M_1, N_1$ | Components in the x-, y-, z-directions of the moment of $F_1$ about the origin of the x-, y-, z-axes |
| $k$ | Factor of proportionality relating the added mass effect to the mass of fluid displaced |
| $k_1, k_2, k'$ | Constants used to evaluate certain of the added mass derivatives of a prolate spheroid when the origin of the body axes is at the center of the spheroid and the x-axis is the axis of revolution for the spheroid |
| $m$ | Actual mass of the body |
| $O$ | Origin of the x, y, z body axes |
| $p, q, r$ | Components in the x-, y-, z-directions of the angular velocity of the body relative to the fluid |
| $T$ | Kinetic energy of the fluid |
| $t$ | Time |
| $u, v, w$ | Components in the x-, y-, z-directions of the linear velocity of the point O relative to the fluid |

| | |
|---|---|
| $X_{\dot{u}}, X_{\dot{v}}, X_{\dot{w}}, X_{\dot{p}}, X_{\dot{q}}, X_{\dot{r}}$<br>$Y_{\dot{v}}, Y_{\dot{w}}, Y_{\dot{p}}, Y_{\dot{q}}, Y_{\dot{r}}$<br>$Z_{\dot{w}}, Z_{\dot{p}}, Z_{\dot{q}}, Z_{\dot{r}}$<br>$K_{\dot{p}}, K_{\dot{q}}, K_{\dot{r}}$<br>$M_{\dot{q}}, M_{\dot{r}}$<br>$N_{\dot{r}}$ | The 21 constants, called added mass derivatives, that characterize the added mass properties of the body. Each derivative has the form indicated by the typical relation $X_{\dot{u}} = \frac{\partial X}{\partial \dot{u}}$ |
| $X_1, Y_1, Z_1$ | Components in the x-, y-, z-directions of $F_1$ |
| x, y, z | A set of moving, orthogonal, right-hand, Cartesian coordinate axes fixed in the body; with the location of the origin O of the axes and the orientation of the axes with respect to the body both arbitrary unless stated otherwise at certain specific points in the text |
| $\alpha_o, \beta_o, \gamma_o$ | Constants that define the relative proportions of an ellipsoid |
| $\rho$ | Mass density of the fluid |
| $\cdot$ | Dot over a quantity. The rate of change of a quantity with respect to time |

## ABSTRACT

Expressions are given for the complete "added mass" effect for any rigid body moving in any manner in an ideal fluid. The expressions give the force and moment acting on the body in terms of 21 added mass derivatives. These derivatives are the maximum number that are independent for a Cartesian set of body axes. Reduced expressions are also given for a finned prolate spheroid with the origin of the body axes located at some point on the axis of revolution. Theoretical values of the added mass derivatives are given for an ellipsoid and a prolate spheroid when the reference axes are principal axes for an origin located at the center of the ellipsoid or spheroid. The added mass effect for a stationary body in an accelerating fluid is also described.

## INTRODUCTION

As part of the Fundamental Hydromechanics Research Program at the David Taylor Model Basin, the general equations of motion of a rigid body partially or totally immersed in a fluid have been under study. These equations contain terms which, for convenience in the present discussion, are referred to as "added mass" terms. The body moving in the fluid behaves as though it has more mass than is actually the case. The apparent increase in mass varies with the kind of motion and the apparent distribution of the added mass also depends upon the nature of the motion. What the added mass phenomenon is, and what terms should be used to represent it in equations of motion do not appear to be understood clearly by many persons. Clarity has not been enhanced, furthermore, by the variety of names, such as "virtual mass," "ascension to mass," "apparent mass," and "hydrodynamic mass" applied to the phenomenon.

Much has been written on the subject of added mass and a number of authors have dealt very effectively with one aspect or another of the matter. A usual characteristic of such treatments, however, is that they are slanted toward some specific application. As a result, when equations of motion are presented they usually contain only those added mass terms that are important for a specialized end use.

Such approximate equations are often used as the starting point when workers attempt to develop equations that will apply to other classes of body, to other dynamic environments, or to the improvement of the prediction of the dynamic behavior of bodies. Using such an approach, one runs the risk of overlooking terms that have been omitted as unimportant for an earlier end use, but may be significant for the new application. The author felt that it might be helpful if the complete expressions for the added mass of a body moving in an ideal fluid were reviewed in studying the general equations of motion.

Examination of several probable sources showed that a general expression is given for the kinetic energy of the fluid surrounding the body, but that the treatment has not been continued to the point of obtaining general expressions for forces and moments arising from the added mass effect.[2,3,4] Accordingly, the necessary simple differentiations were carried out and the results are presented here in the hope that they may have reference value.

The paper is arranged with the general equations first because easy access to them seemed most important. They are followed by expressions for the added mass of a finned prolate spheroid with the origin of the reference axes located somewhere on the axis of revolution of the spheroid. These reduced equations serve to highlight the most important added mass terms for the elongated finned bodies of revolution that are often used as underwater vehicles. Following the presentation of the added mass equations is a brief discussion of the method of derivation and some of their salient features. Finally, some remarks are made on the added mass of a stationary body in an accelerating fluid.

## COMPLETE EXPRESSIONS FOR ADDED MASS

The equations of motion for a rigid body customarily are referred to a set of moving orthogonal Cartesian axes fixed in the body. If the body is partially or totally immersed in a fluid, which would be at rest except for the motion of the body, added mass terms usually appear as part of such equations of motion. When the motion of the body is completely general, and there is no restriction on the shape of the body or the location of the axes with respect to the body, the complete expressions for the added mass in a frictionless fluid are:

$$
\begin{aligned}
X_1 = {} & X_{\dot{u}}\dot{u} + X_{\dot{w}}(\dot{w} + uq) + X_{\dot{q}}\dot{q} + Z_{\dot{w}}wq + Z_{\dot{q}}q^2 \\
& + X_{\dot{v}}\dot{v} + X_{\dot{p}}\dot{p} + X_{\dot{r}}\dot{r} - Y_{\dot{v}}vr - Y_{\dot{p}}rp - Y_{\dot{r}}r^2 \\
& - X_{\dot{v}}ur - Y_{\dot{w}}wr \\
& + Y_{\dot{w}}vq + Z_{\dot{p}}pq - (Y_{\dot{q}} - Z_{\dot{r}})qr
\end{aligned}
\qquad [1a]
$$

[2]References are listed on page 19.

$$
\begin{aligned}
Y_1 = {} & X_{\dot v}\dot u + Y_{\dot w}\dot w + Y_{\dot q}\dot q \\
& + Y_{\dot v}\dot v + Y_{\dot p}\dot p + Y_{\dot r}\dot r + X_{\dot v} vr - Y_{\dot w} vp + X_{\dot r} r^2 + (X_{\dot p} - Z_{\dot r}) rp - Z_{\dot p} p^2 \\
& - X_{\dot w} (up - wr) + X_{\dot u} ur - Z_{\dot w} wp \\
& - Z_{\dot q} pq + X_{\dot q} qr
\end{aligned}
\qquad [1b]
$$

$$
\begin{aligned}
Z_1 = {} & X_{\dot w}(\dot u - wq) + Z_{\dot w}\dot w + Z_{\dot q}\dot q - X_{\dot u} uq - X_{\dot q} q^2 \\
& + Y_{\dot w}\dot v + Z_{\dot p}\dot p + Z_{\dot r}\dot r + Y_{\dot v} vp + Y_{\dot r} rp + Y_{\dot p} p^2 \\
& + X_{\dot v} up + Y_{\dot w} wp \\
& - X_{\dot v} vq - (X_{\dot p} - Y_{\dot q}) pq - X_{\dot r} qr
\end{aligned}
\qquad [1c]
$$

$$
\begin{aligned}
K_1 = {} & X_{\dot p}\dot u + Z_{\dot p}\dot w + K_{\dot q}\dot q - X_{\dot v} wu + X_{\dot r} uq - Y_{\dot w} w^2 - (Y_{\dot q} - Z_{\dot r}) wq + M_{\dot r} q^2 \\
& + Y_{\dot p}\dot v + K_{\dot p}\dot p + K_{\dot r}\dot r + Y_{\dot w} v^2 - (Y_{\dot q} - Z_{\dot r}) vr + Z_{\dot p} vp - M_{\dot r} r^2 - K_{\dot q} rp \\
& + X_{\dot w} uv - (Y_{\dot v} - Z_{\dot w}) vw - (Y_{\dot r} + Z_{\dot q}) wr - Y_{\dot p} wp - X_{\dot q} ur \\
& + (Y_{\dot r} + Z_{\dot q}) vq + K_{\dot r} pq - (M_{\dot q} - N_{\dot r}) qr
\end{aligned}
\qquad [1d]
$$

$$
\begin{aligned}
M_1 = {} & X_{\dot q} (\dot u + wq) + Z_{\dot q}(\dot w - uq) + M_{\dot q}\dot q - X_{\dot w} (u^2 - w^2) - (Z_{\dot w} - X_{\dot u}) wu \\
& + Y_{\dot q}\dot v + K_{\dot q}\dot p + M_{\dot r}\dot r + Y_{\dot p} vr - Y_{\dot r} vp - K_{\dot r}(p^2 - r^2) + (K_{\dot p} - N_{\dot r}) rp \\
& - Y_{\dot w} uv + X_{\dot v} vw - (X_{\dot r} + Z_{\dot p})(up - wr) + (X_{\dot p} - Z_{\dot r})(wp + ur) \\
& - M_{\dot r} pq + K_{\dot q} qr
\end{aligned}
\qquad [1e]
$$

$$
\begin{aligned}
N_1 = {} & X_{\dot r}\dot u + Z_{\dot r}\dot w + M_{\dot r}\dot q + X_{\dot v}u^2 + Y_{\dot w}wu - (X_{\dot p} - Y_{\dot q})uq - Z_{\dot p}wq - K_{\dot q}q^2 \\
& + Y_{\dot r}\dot v + K_{\dot r}\dot p + N_{\dot r}\dot r - X_{\dot v}v^2 - X_{\dot r}vr - (X_{\dot p} - Y_{\dot q})vp + M_{\dot r}rp + K_{\dot q}p^2 \\
& - (X_{\dot u} - Y_{\dot v})uv - X_{\dot w}vw + (X_{\dot q} + Y_{\dot p})up + Y_{\dot r}ur + Z_{\dot q}wp \\
& - (X_{\dot q} + Y_{\dot p})vq - (K_{\dot p} - M_{\dot q})pq - K_{\dot r}qr
\end{aligned}
\qquad [1f]
$$

Definitions of the various symbols used in Equations [1] are given in the Notation. Note that the various linear and angular velocities u, v, w, p, q, r are those of the body relative to the stationary fluid - furthermore they are actual velocities, not incremental variations from an equilibrium condition. Time derivatives, such as $\dot u$, are relative to the moving x-, y-, z-axes. The quantities $X_1$, $Y_1$, $Z_1$ are components in the x-, y-, z-directions of the reaction force $F_1$ that is exerted on the body by the fluid during accelerated motion. Quantities $K_1$, $M_1$, $N_1$ are the x-, y-, z-components of the moment of $F_1$ about the origin O of the coordinate system. As thus defined, $F_1$ is part of the total external force F acting on the body.

Equations [1] are in six parts. Each part is arranged with longitudinal components of the motion on the first line and lateral components on the second line. The third line contains mixed terms involving the u or w velocity as one factor. Often one or both of these velocities are large enough to be treated as constants during the motion, thus permitting the affected terms on the third line to be treated as additional terms in the lateral components of motion. The fourth line contains mixed components of motion of such a nature that they usually can be neglected as second order terms.

The general expressions for added mass, represented by Equations [1], contain 21 constants (of which $X_{\dot u}$ is typical). This is the maximum number of such constants that are independent for the Cartesian reference frame used. In principle, one can write 36 constants, relating the 6 components of force and moment to the velocity derivatives in the 6 degrees of freedom, as shown in the following array:

$$
\begin{array}{cccccc}
X_{\dot{u}} & X_{\dot{v}} & X_{\dot{w}} & X_{\dot{p}} & X_{\dot{q}} & X_{\dot{r}} \\
Y_{\dot{u}} & Y_{\dot{v}} & Y_{\dot{w}} & Y_{\dot{p}} & Y_{\dot{q}} & Y_{\dot{r}} \\
Z_{\dot{u}} & Z_{\dot{v}} & Z_{\dot{w}} & Z_{\dot{p}} & Z_{\dot{q}} & Z_{\dot{r}} \\
K_{\dot{u}} & K_{\dot{v}} & K_{\dot{w}} & K_{\dot{p}} & K_{\dot{q}} & K_{\dot{r}} \\
M_{\dot{u}} & M_{\dot{v}} & M_{\dot{w}} & M_{\dot{p}} & M_{\dot{q}} & M_{\dot{r}} \\
N_{\dot{u}} & N_{\dot{v}} & N_{\dot{w}} & N_{\dot{p}} & N_{\dot{q}} & N_{\dot{r}}
\end{array}
$$

In a real fluid these 36 constants may all be distinct. With ideal (frictionless) fluid, however, the constants that are symmetrical with respect to the indicated diagonal of the array are equal. Thus $Y_{\dot{u}} \equiv X_{\dot{v}}$, $M_{\dot{w}} \equiv Z_{\dot{q}}$, etc. As indicated, these relations are identities, independent of the nature of the body. Consequently, it is sufficient to retain only the constants on and above the diagonal in the array, thus:

$$
\begin{array}{cccccc}
X_{\dot{u}} & X_{\dot{v}} & X_{\dot{w}} & X_{\dot{p}} & X_{\dot{q}} & X_{\dot{r}} \\
 & Y_{\dot{v}} & Y_{\dot{w}} & Y_{\dot{p}} & Y_{\dot{q}} & Y_{\dot{r}} \\
 & & Z_{\dot{w}} & Z_{\dot{p}} & Z_{\dot{q}} & Z_{\dot{r}} \\
 & & & K_{\dot{p}} & K_{\dot{q}} & K_{\dot{r}} \\
 & & & & M_{\dot{q}} & M_{\dot{r}} \\
 & & & & & N_{\dot{r}}
\end{array}
$$

The constants, which will be called added mass derivatives, are functions only of the body shape and the density of the fluid. The 21 added mass derivatives are necessary and sufficient to define completely the added mass properties of any body moving in any manner in an ideal fluid. Experience has shown that the numerical values of added mass in a real fluid usually are in good agreement with those obtained from ideal fluid theory.[5]

## EXPRESSIONS FOR ADDED MASS OF A FINNED PROLATE SPHEROID

Many of the 21 added mass derivatives contained in the general expressions for added mass are either zero or mutually related when the body has various symmetries. Bodies of practical interest almost invariably have a vertical plane of symmetry. Bodies used as underwater vehicles usually are even more symmetrical and a finned prolate spheroid is a good approximation to many such bodies.

Assume a prolate spheroid with the origin of a Cartesian set of body axes located at any arbitrary point on the axis of revolution, which is also the x-axis. Mutually perpendicular y- and z-axes will also be perpendicular to the x-axis. Assume that the spheroid is fitted with a dorsal fin near the maximum diameter of the body and lying in the xz-plane. In addition, assume that the body has four fins that are identical and are located at the rear of the body. Let these four fins each have the same x-coordinate and be placed in cruciform fashion around the x-axis. Furthermore, let the plane of each fin pass through the x-axis. This finned body can provide a rather good approximation to a submarine or airship, as far as the theoretical values for added mass is concerned, when the spheroid has the same fineness ratio and displacement as the actual body.[6]

For such a finned body the general expressions become much simpler, namely:

$$X_1 = X_{\dot{u}}\dot{u} - Y_{\dot{v}}rv + Z_{\dot{w}}qw - Y_{\dot{r}}r^2 + Z_{\dot{q}}q^2 - Y_{\dot{p}}rp \tag{2a}$$

$$Y_1 = X_{\dot{u}}ru + Y_{\dot{v}}\dot{v} - Z_{\dot{w}}pw + Y_{\dot{r}}\dot{r} - Z_{\dot{q}}pq + Y_{\dot{p}}\dot{p} \tag{2b}$$

$$Z_1 = -X_{\dot{u}}qu + Y_{\dot{v}}pv + Z_{\dot{w}}\dot{w} + Y_{\dot{r}}pr + Z_{\dot{q}}\dot{q} + Y_{\dot{p}}p^2 \tag{2c}$$

$$\begin{aligned} K_1 = &- Y_{\dot{v}}wv + Z_{\dot{w}}vw + Y_{\dot{r}}(vq - wr) + Z_{\dot{q}}(vq - wr) - M_{\dot{q}}rq + N_{\dot{r}}qr \\ &+ Y_{\dot{p}}(\dot{v} - wp) + K_{\dot{p}}\dot{p} + K_{\dot{r}}(\dot{r} + qp) \end{aligned} \tag{2d}$$

$$\begin{aligned} M_1 = &(X_{\dot{u}} - Z_{\dot{w}})uw - Y_{\dot{r}}pv + Z_{\dot{q}}(\dot{w} - uq) + M_{\dot{q}}\dot{q} - N_{\dot{r}}pr \\ &+ Y_{\dot{p}}rv + K_{\dot{p}}rp + K_{\dot{r}}(r^2 - p^2) \end{aligned} \tag{2e}$$

$$N_1 = -(X_{\dot{u}} - Y_{\dot{v}})vu + Y_{\dot{r}}(\dot{v} + ur) + Z_{\dot{q}}pw + M_{\dot{q}}pq + N_{\dot{r}}\dot{r}$$

$$+ Y_{\dot{p}}(up - vq) - K_{\dot{p}}qp + K_{\dot{r}}(\dot{p} - qr) \qquad [2f]$$

The remarks on notation following Equations [1] also apply to Equations [2]. The reduced complexity of Equations [2] results from the symmetry of the finned spheroid, whereby

$$\left.\begin{aligned} X_{\dot{v}} = X_{\dot{w}} = X_{\dot{p}} = X_{\dot{q}} = X_{\dot{r}} &= 0 \\ Y_{\dot{w}} = Y_{\dot{q}} &= 0 \\ Z_{\dot{p}} = Z_{\dot{r}} &= 0 \\ K_{\dot{q}} &= 0 \\ M_{\dot{r}} &= 0 \end{aligned}\right\} \qquad [3]$$

The order of the terms has been rearranged in Equations [2] from what it was in Equations [1] so terms involving derivatives of small magnitude are at the end, and derivatives of nearly equal magnitude are grouped together.

## ADDED MASS FOR A BODY OF REVOLUTION WITH SYMMETRIC FINS

Removal of the dorsal fin would make $Y_{\dot{p}}$ and $K_{\dot{r}}$ zero and reduce the magnitude of $K_{\dot{p}}$ in Equations [2]; also, other derivatives that are nearly equal in magnitude would become equal, namely:

$$\left.\begin{aligned} Y_{\dot{v}} &= Z_{\dot{w}} \\ Y_{\dot{r}} &= -Z_{\dot{q}} \\ M_{\dot{q}} &= N_{\dot{r}} \end{aligned}\right\} \qquad [4]$$

Hence, for a body of revolution with symmetric fins only, Equations [2] reduce to

$$X_1 = X_{\dot{u}}\dot{u} - Y_{\dot{v}}(rv - qw) - Y_{\dot{r}}(q^2 + r^2) \qquad [5a]$$

$$Y_1 = X_{\dot{u}}ru + Y_{\dot{v}}(\dot{v} - pw) + Y_{\dot{r}}(\dot{r} + pq) \qquad [5b]$$

$$Z_1 = - X_{\dot{u}}qu + Y_{\dot{v}}(\dot{w} + pv) - Y_{\dot{r}}(\dot{q} - pr) \qquad [5c]$$

$$K_1 = K_{\dot{p}}\dot{p} \qquad [5d]$$

$$M_1 = (X_{\dot{u}} - Y_{\dot{v}})uw - Y_{\dot{r}}(\dot{w} + pv - qu) + N_{\dot{r}}\dot{q} - (N_{\dot{r}} - K_{\dot{p}})pr \qquad [5e]$$

$$N_1 = - (X_{\dot{u}} - Y_{\dot{v}})vu + Y_{\dot{r}}(\dot{v} + ru - pw) + N_{\dot{r}}\dot{r} + (N_{\dot{r}} - K_{\dot{p}})pq \qquad [5f]$$

Normally $K_{\dot{p}}$ would be negligible compared to $N_{\dot{r}}$ in Equations [5e] and [5f]. If there were no fins, $K_{\dot{p}}$ would be zero and with the usual sizes of tail fin it probably can be approximated safely by zero in Equation [5d].

## DISCUSSION OF THE ADDED MASS EXPRESSIONS

The derivation of the added mass expressions usually is based on energy or momentum considerations. In this paper the energy approach will be followed.

### EXPLANATION OF KINETIC ENERGY IN FLUID

The kinetic energy in the fluid will be described first for steady motion, and then for accelerated motion, of the body.

#### STEADY MOTION

Equations of motion for a body moving in a fluid are basically a statement of Newton's Second Law

$$F = ma \qquad [6]$$

where F is the total external force on the body, m the actual mass of the body, and a its linear acceleration in the direction of F.

Consider first the case where the motion is steady and the body is either partially or totally immersed in fluid. The steady motion implies zero acceleration, so the tôtal external force F is zero. It is well known that even during such steady motion, in a real fluid, certain hydrodynamic reactions of the fluid produce external forces on the body. An example is the resistance, or drag, force. If the motion is to be steady, such hydrodynamic forces must be balanced by other forces - the resistance, for example, is balanced by a propulsive force. The treatment, in the equations of motion, of the hydrodynamic components of force pertaining to steady motion seems to be readily grasped by most students of the subject.

Because of the presence of the fluid around the body, however, another action takes place that seems to be less well understood. Any motion of the body induces a motion in the otherwise stationary fluid because the fluid must move aside and then close in behind the body in order that the body may make a passage through the fluid. As a consequence, the fluid possesses kinetic energy that it would lack if the body were not in motion. The body has to impart the kinetic energy to the fluid by doing work on the fluid, i.e., the work done is equal to the change in kinetic energy. Any adequate equations of motion for the body must take into account this kinetic energy given to the fluid by the body. This is the function of the added mass terms in the equations.

When the motion of the body is steady, the corresponding fluid motion is steady and the kinetic energy in the fluid is constant. Hence no work is done on the fluid as long as the motion of the body remains steady. It follows that if only steady motions are to be studied, the added mass terms can be omitted in the equations of motion.

## ACCELERATED MOTION

The discussion will now be turned to accelerated motion of the body. Many so-called steady motions are actually accelerated motions, the most important example being the steady turn where the body must continually be accelerated inward in order to pursue a curved path. Returning to Equation [6], assume that F has a value other than zero. Then, according to Equation [6], the body will be accelerated. A change in motion of the body cannot occur, however, without a related change in the state of motion of the surrounding fluid. The change in motion of the fluid will result in a change in the kinetic energy of the fluid. In other words, in order to accelerate, the body must do work on the fluid.

Work is accomplished by moving a force through a distance. In this case the distance is that through which the body moves, and the force is the summation over the body surface of the pressure that the body exerts on the fluid. In applying a force to the fluid, the body experiences an opposite reaction force $F_1$, which is part of the total external force $F$. Let the remainder of the contributions to $F$ be called $F_2$. Then Equation [6] takes the form

$$F_1 + F_2 = m\dot{a} \qquad [7]$$

Note that vector addition is involved in Equation [7]. In Figure 1, suppose that a sphere is immersed in a fluid and accelerated to the right. Then the reaction force $F_1$ is directed to the left, and $F_2$ must act in the opposite direction and be large enough to overcome $F_1$ and also accelerate the sphere.

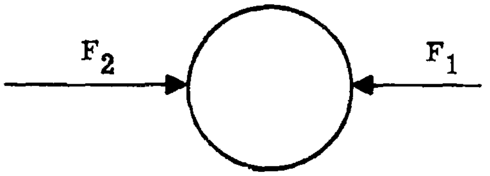


Figure 1 - External Forces on an Accelerating Sphere

## ADDED MASS EFFECT IN TERMS OF KINETIC ENERGY

In general the force $F_1$ will have components $X_1$, $Y_1$, $Z_1$ along the x, y, z body axes and moment components $K_1$, $M_1$, $N_1$ about the axes. On pages 168 and 169, Equations (2) and (3), of Reference 2, Lamb gives expressions that may be used to obtain these components when the kinetic energy of the fluid is varied. The desired relations are:

$$\left.\begin{aligned}
X_1 &= -\frac{d}{dt}\frac{\partial T}{\partial u} - q\frac{\partial T}{\partial w} + r\frac{\partial T}{\partial v} \\
Y_1 &= -\frac{d}{dt}\frac{\partial T}{\partial v} - r\frac{\partial T}{\partial u} + p\frac{\partial T}{\partial w} \\
Z_1 &= -\frac{d}{dt}\frac{\partial T}{\partial w} - p\frac{\partial T}{\partial v} + q\frac{\partial T}{\partial u} \\
K_1 &= -\frac{d}{dt}\frac{\partial T}{\partial p} - q\frac{\partial T}{\partial r} + r\frac{\partial T}{\partial q} - v\frac{\partial T}{\partial w} + w\frac{\partial T}{\partial v} \\
M_1 &= -\frac{d}{dt}\frac{\partial T}{\partial q} - r\frac{\partial T}{\partial p} + p\frac{\partial T}{\partial r} - w\frac{\partial T}{\partial u} + u\frac{\partial T}{\partial w} \\
N_1 &= -\frac{d}{dt}\frac{\partial T}{\partial r} - p\frac{\partial T}{\partial q} + q\frac{\partial T}{\partial p} - u\frac{\partial T}{\partial v} + v\frac{\partial T}{\partial u}
\end{aligned}\right\} \qquad [8]$$

where T is the total kinetic energy of the fluid and t is time.

## GENERAL EXPRESSION FOR KINETIC ENERGY IN FLUID

On page 163, loc. cit., Lamb gives the following expression for twice the total kinetic energy of the fluid:

$$\begin{aligned}
2T = &- X_{\dot{u}}u^2 - Y_{\dot{v}}v^2 - Z_{\dot{w}}w^2 - 2Y_{\dot{w}}vw - 2X_{\dot{w}}wu - 2X_{\dot{v}}uv \\
&- K_{\dot{p}}p^2 - M_{\dot{q}}q^2 - N_{\dot{r}}r^2 - 2M_{\dot{r}}qr - 2K_{\dot{r}}rp - 2K_{\dot{q}}pq \\
&- 2p(X_{\dot{p}}u + Y_{\dot{p}}v + Z_{\dot{p}}w) \\
&- 2q(X_{\dot{q}}u + Y_{\dot{q}}v + Z_{\dot{q}}w) \\
&- 2r(X_{\dot{r}}u + Y_{\dot{r}}v + Z_{\dot{r}}w)
\end{aligned} \qquad [9]$$

where Lamb's notation has been replaced by that of Reference 1. Equation [9] is general and applies to any body moving in a fluid. It contains the same 21 independent added mass derivatives found in Equations [1].

## DERIVATIVES OF KINETIC ENERGY WITH RESPECT TO FLUID VELOCITIES

Six partial derivatives must be obtained from Equation [9] in order to expand Equations [8], namely:

$$\left.\begin{aligned}
\frac{\partial T}{\partial u} &= - X_{\dot u} u - X_{\dot w} w - X_{\dot v} v - X_{\dot p} p - X_{\dot q} q - X_{\dot r} r \\
\frac{\partial T}{\partial v} &= - Y_{\dot v} v - Y_{\dot w} w - X_{\dot v} u - Y_{\dot p} p - Y_{\dot q} q - Y_{\dot r} r \\
\frac{\partial T}{\partial w} &= - Z_{\dot w} w - Y_{\dot w} v - X_{\dot w} u - Z_{\dot p} p - Z_{\dot q} q - Z_{\dot r} r \\
\frac{\partial T}{\partial p} &= - K_{\dot p} p - K_{\dot r} r - K_{\dot q} q - X_{\dot p} u - Y_{\dot p} v - Z_{\dot p} w \\
\frac{\partial T}{\partial q} &= - M_{\dot q} q - M_{\dot r} r - K_{\dot q} p - X_{\dot q} u - Y_{\dot q} v - Z_{\dot q} w \\
\frac{\partial T}{\partial r} &= - N_{\dot r} r - M_{\dot r} q - K_{\dot r} p - X_{\dot r} u - Y_{\dot r} v - Z_{\dot r} w
\end{aligned}\right\} \qquad [10]$$

Substitution of Equations [10] into Equations [8] will yield Equations [1].

## ADDED MASS GOVERNED BY FLOW PATTERN, NOT ACCELERATION

It seems desirable to point out that the same acceleration, as measured along the body axes, can be produced by different types of motion of the body. The added mass effect does not depend on the acceleration, per se, but on the nature of the body motion, so the added mass effect may differ under two sets of circumstances where the acceleration is the same. For example, a prolate spheroid, moving in a perfect fluid in the direction of the x-axis, and experiencing an acceleration $\dot u$, would have streamlines of the sort sketched in Figure 2.

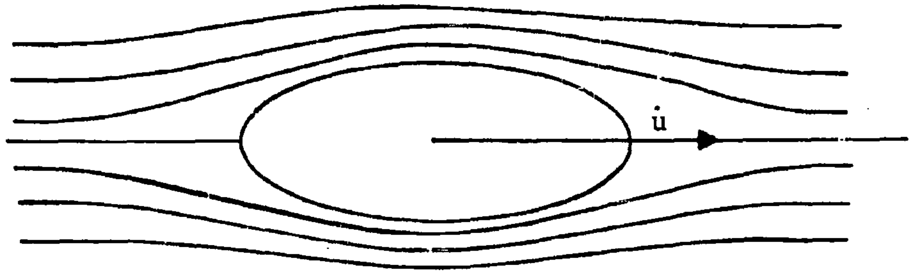


Figure 2 - Streamlines for Spheroid Accelerated Along x-Axis

Consider now the same spheroid making a steady turn at a fixed angle of drift. The nature of the potential flow would be that shown in Figure 3.

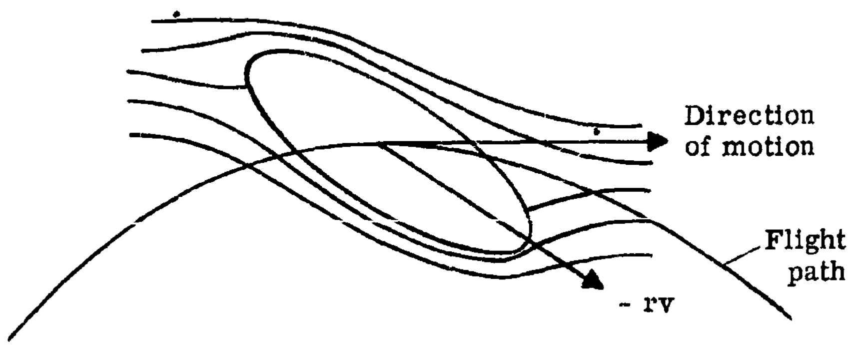


Figure 3 - Streamlines for a Spheroid in a Steady Turn

Although all the velocities are steady in Figure 3, the Coriolis acceleration[7] produces an acceleration of the body of the amount $-rv$ along the x-axis. The actual mass m of the body would react in the same way to either the $\dot{u}$ of Figure 2 or the $-rv$ of Figure 3.

The added mass effect would definitely be different in the two cases, however, because of the difference in the flow patterns attending the two different motions of the body.[8] Recalling that a given flow pattern requires a unique velocity potential to produce it (see Reference 4, Sec. 3.71, p. 87); that the kinetic energy is derivable from the velocity potential (see, for example, Equation (1), Sec. 121, p. 163, of Reference 2); and that the added mass effects are the result of changes in the kinetic energy of the fluid (Equations [8]); it is reasonable to expect different added mass effects from the two different flow patterns.

## DIFFERENT EXPERIMENTAL APPROACHES

The preceding discussion suggests that a given added mass derivative might be evaluated experimentally by two or more basically different techniques. As an example, $Y_{\dot{v}}$ could be determined, in principle, by either accelerating the body in the direction of the y-axis or by giving the body the motion depicted in Figure 3. The author believes that if experiments were conducted with perfection, using two different modes of motion in a real fluid, the resultant values of $Y_{\dot{v}}$ would differ from each other and from the theoretical value because the two motions differ in the degree to which the conditions of potential flow are met. It seems desirable, however, to adopt the basic added mass expressions in the form presented in Equations [1], which applies to potential flow conditions, and to add corrective second or higher order terms to the basic equations if sufficient experimental data are available to make such corrections feasible.

## NATURE OF ADDED MASS DERIVATIVES

The purpose of the following discussion is twofold: to show the relation between the geometry of the body and the values of its theoretical added mass derivatives; and to demonstrate that the added mass is related not to the mass of the body but to the mass of the fluid displaced.

### SOURCES OF NUMERICAL DATA

The intended scope of this paper does not include a discussion of methods of calculating or measuring the values of the 21 added mass derivatives that characterize the added mass properties of a body. Evaluation of certain of the derivatives has been treated by many authors. A few representative papers are those of References 5, 6, and 9 through 17. Most of the titles are adequately descriptive of the material treated, but it should be mentioned that Reference 5 covers a wide range of valuable background material. References 13 and 14 contain experimental data on rectangular plates and prisms. The theoretical values of the derivatives pertaining to ellipsoid and a prolate spheroid will be examined in some detail at this point in order to indicate the nature of the added mass derivatives.

### ADDED MASS DERIVATIVES FOR AN ELLIPSOID

The equation of an ellipsoid, with elliptical cross sections in all three planes of symmetry, when referred to principal axes with the origin at the center of the ellipsoid, is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0 \qquad [11]$$

**where a, b, and c are the semiaxes. On page 164 of Reference 2, Lamb gives for such an ellipsoid**

$$
\left.
\begin{aligned}
X_{\dot{u}} &= - \frac{\alpha_0}{2 - \alpha_0} \frac{4}{3} \pi \rho a b c \\
Y_{\dot{v}} &= - \frac{\beta_0}{2 - \beta_0} \frac{4}{3} \pi \rho a b c \\
Z_{\dot{w}} &= - \frac{\gamma_0}{2 - \gamma_0} \frac{4}{3} \pi \rho a b c \\
K_{\dot{p}} &= - \frac{1}{5} \frac{(b^2 - c^2)^2 (\gamma_0 - \beta_0)}{2(b^2 - c^2) + (b^2 + c^2)(\beta_0 - \gamma_0)} \frac{4}{3} \pi \rho a b c \\
M_{\dot{q}} &= - \frac{1}{5} \frac{(c^2 - a^2)^2 (\alpha_0 - \gamma_0)}{2(c^2 - a^2) + (c^2 + a^2)(\gamma_0 - \alpha_0)} \frac{4}{3} \pi \rho a b c \\
N_{\dot{r}} &= - \frac{1}{5} \frac{(a^2 - b^2)^2 (\beta_0 - \alpha_0)}{2(a^2 - b^2) + (a^2 + b^2)(\alpha_0 - \beta_0)} \frac{4}{3} \pi \rho a b c
\end{aligned}
\right\} \qquad [12]
$$

where $\alpha_0$, $\beta_0$, $\gamma_0$ are constants that describe the relative proportions of the ellipsoid (see p. 153, loc. cit.). Also

$$
\left.
\begin{aligned}
& X_{\dot{v}} = X_{\dot{w}} = X_{\dot{p}} = X_{\dot{q}} = X_{\dot{r}} = Y_{\dot{w}} = Y_{\dot{p}} = Y_{\dot{q}} = Y_{\dot{r}} = 0 \\
& Z_{\dot{p}} = Z_{\dot{q}} = Z_{\dot{r}} = K_{\dot{q}} = K_{\dot{r}} = M_{\dot{r}} = 0
\end{aligned}
\right\} \qquad [13]
$$

from Equations [12] it is evident that the added mass derivatives are functions only of the shape and size of the ellipsoid and of the fluid density $\rho$.

## ADDED MASS DERIVATIVES FOR A PROLATE SPHEROID

A prolate spheroid revolved about the x-axis is obtained from the ellipsoid of Equation [11] if $b = c$ and $a > b$. Then $\beta_0 = \gamma_0$ and, from

Equations [12], $Y_{\dot{v}} = Z_{\dot{w}}$; also $M_{\dot{q}} = N_{\dot{r}}$ and $K_{\dot{p}} = 0$. Equations [13] are still valid. Consequently,

$$\left.\begin{aligned}
X_{\dot{u}} &= - \frac{\alpha_0}{2 - \alpha_0} \, \frac{4}{3} \pi \rho \, ab^2 \\
Y_{\dot{v}} &= Z_{\dot{w}} = - \frac{\beta_0}{2 - \beta_0} \, \frac{4}{3} \pi \rho \, ab^2 \\
K_{\dot{p}} &= 0 \\
N_{\dot{r}} &= M_{\dot{q}} = - \frac{1}{5} \, \frac{(b^2 - a^2)^2 (\alpha_0 - \beta_0)}{2(b^2 - a^2) + (b^2 + a^2)(\beta_0 - \alpha_0)} \, \frac{4}{3} \pi \rho ab^2
\end{aligned}\right\} \qquad [14]$$

For the prolate spheroid under discussion

$$e^2 = 1 - (b/a)^2 \qquad [15]$$

where e is the eccentricity of the meridian elliptical section. Also

$$\left.\begin{aligned}
\alpha_0 &= \frac{2(1-e^2)}{e^3} \left( \frac{1}{2} \log \frac{1 + e}{1 - e} - e \right) \\
\beta_0 &= \frac{1}{e^2} - \frac{1 - e^2}{2e^3} \log \frac{1 + e}{1 - e}
\end{aligned}\right\} \qquad [16]$$

LAMB'S k FACTORS

$$\left.\begin{aligned}
\text{Let} \quad k_1 &= \frac{\alpha_0}{2 - \alpha_0} \\
k_2 &= \frac{\beta_0}{2 - \beta_0} \\
k' &= \frac{e^4 (\beta_0 - \alpha_0)}{(2 - e^2) \left[ 2e^2 - (2 - e^2)(\beta_0 - \alpha_0) \right]}
\end{aligned}\right\} \qquad [17]$$

Then Equations [14] may be written

$$\left.\begin{aligned} X_{\dot{u}} &= -k_1 \frac{4}{3}\pi\rho ab^2 \\ Y_{\dot{v}} = Z_{\dot{w}} &= -k_2 \frac{4}{3}\pi\rho ab^2 \\ N_{\dot{r}} = M_{\dot{q}} &= -k' \frac{4}{15}\pi\rho ab^2 (a^2 + b^2) \end{aligned}\right\} \quad [18]$$

All the remaining added mass derivatives are zero for the prolate spheroid (for the assumed location of the reference axes). By virtue of Equations [15], [16], and [17], the values of $k_1$, $k_2$, and k' are pure numbers determined solely by the fineness ratio a/b of the spheroid. Representative values of the three parameters are tabulated on page 155 of Reference 2.

## SOME ERRONEOUS CONCEPTS

In Equations [18], the factor $\frac{4}{3}\pi\rho ab^2$ is the mass of the volume of fluid displaced by the spheroid and $\frac{4}{15}\pi\rho ab^2 (a^2 + b^2)$ is the moment of inertia, about the y- or z-axis, of the same volume of fluid. Although Equations [18] show that the added mass derivatives are proportional to the mass or moment of inertia of this specific volume of fluid, the mistake must not be made of assuming that the added mass effects involve only a limited volume of the fluid, or that some limited volume of fluid moves with the body - the so-called "entrained fluid" erroneously described by some authors. All the particles of fluid move, although the motion of the fluid is more pronounced in the neighborhood of the body. Darwin has endeavored to describe the nature of this motion. [18]

The preceding discussion established that the added mass derivatives for a prolate spheroid have the dimensions of either a mass or a moment of inertia. This result is generally true for any body - hence the added mass derivatives for any body can always be expressed as some proportionality factor times either the mass of fluid displaced by the body or some moment of inertia of that configuration of fluid.

## FIXED BODY IN AN ACCELERATING FLUID

The main purpose of this paper is to present expressions for a body accelerated in a fluid that is otherwise at rest. A few remarks are considered desirable, however, on the different added mass effect experienced if the body is stationary and the fluid is accelerated. The difference arises because of the pressure gradient required in the fluid in order to accelerate the fluid flow.

Euler's equation shows that the pressure gradient in an accelerating fluid is opposite to the direction of the acceleration and its magnitude is the product of the fluid density and the acceleration. When the acceleration is uniform throughout a region of fluid a uniform pressure gradient exists.

The effect of an uniform gradient in producing a force on a body is demonstrated by considering the uniform pressure gradient that exists in a stationary fluid resting in the earth's gravitational field. That gradient produces the well-known buoyancy force, in a direction opposite to the pressure gradient, on any body that displaces some of the fluid. The magnitude of the buoyancy force is given by the product of the volume of fluid displaced times the pressure gradient.

It follows that a fixed body in an uniformly accelerating fluid will experience a buoyancy-type force in the direction of the fluid acceleration. The magnitude of the force is given by the triple product: the fluid density times the volume of fluid displaced times the acceleration of the fluid. This force must be added to the usual force produced by the added mass effect attending the relative acceleration between the body and the fluid.

In the preceding section of this paper, where the nature of the added mass derivatives was discussed, it was stated that the force produced by the added mass effect of an accelerating body in a stationary fluid is proportional to the mass of fluid displaced. If the factor of proportionality is called k, then the force produced by the added mass effect for the reverse situation of a stationary body and an accelerating fluid is obtained by replacing k by $(1 + k)$. Some treatments of added mass in accelerated flows are contained in References 19 and 20.

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